

Finding the Center of Gravity of the Forearm

A typical first year physics course teaches students about the center of gravity using both regular and irregularly shaped objects. Students often suspend an object in each of two dimensions from a string with the intersection determining the center of gravity of the object. While these methods can be effective, they are not particularly exciting or motivating. Wouldn't it be more interesting to relate the center of gravity to your body? Through a series of simple measurements and assumptions, students can calculate the center of gravity of their forearm. Comparisons can be made between the muscular forearm of the football player in the class with the scrawniest arm in the room, creating a deep understanding of center of gravity and how it relates to the human body.

Develop the Idea:

Challenge your students to define a center of gravity and how they would calculate its location before you begin to discuss it as a class. Have each student write down their ideas for two or three minutes. Pair students and let them discuss their ideas for two or three more minutes before soliciting ideas from the class.

Cutnell and Johnson¹ define center of gravity of a rigid object as "the point at which its weight can be considered to act when calculating the torque due to the weight." For a symmetric shape with uniform weight distribution, we know that the center of gravity is simply at the geometric center.

Ask your students to balance a ruler on their finger. Where does it balance? At the center. Now add a small object such as a penny somewhere on the ruler and balance the ruler again. Where does it balance? Somewhere closer to the added object. If you know the mass of the rulers and the added objects, you may wish to calculate the center of gravity with the following equations:

$$W_1x_1 + W_2x_2 = (W_1 + W_2)x_{cog} \quad (1)$$

$$\frac{W_1x_1 + W_2x_2}{(W_1 + W_2)} = x_{cog} \quad (2)$$

where the W variables represent the weights and x variables represent the distance from an axis of rotation as shown in Figure 1.

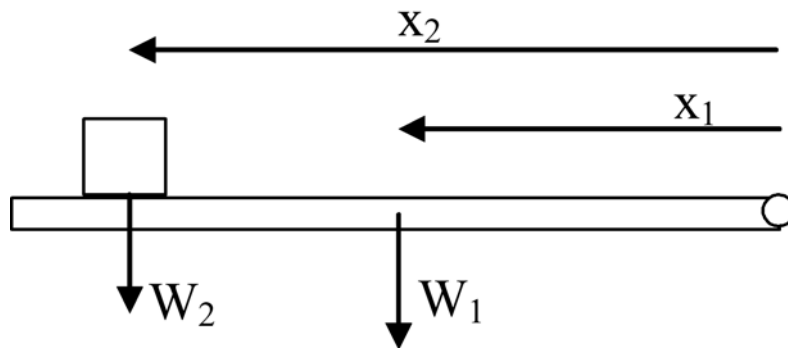


Figure 1. Free body diagram of an object placed on a ruler.

Once students have a basic understanding of center of gravity, ask them to determine a method for finding the center of gravity's location in the forearm.

Procedure:

Begin by pairing students and having one person bend his or her elbow at a ninety degree angle. Use a ruler and mark with a water-soluble pen a dot every centimeter from the crease of the elbow joint to the wrist. At each of these dots, measure and record in a spreadsheet the circumference (C_i) of the arm. Assuming the arm has a circular cross section, calculate the radius (R_i) of the arm from its circumference at each location. This radius can then be used to calculate the cross-sectional area of the arm and then the volume (V_i) of that segment as shown in Figure 2. Each volume can then weighted by the distance it is from the elbow joint, just as it was in equation 2. The center of gravity of the forearm as a whole can be computed from the weight of each segment and its distance from the elbow, similar to equation 2. If the density of each of N segments is assumed to be the same, then the center of gravity can be computed from the segment positions and their volumes:

$$x_{cog} = \frac{\sum_{i=1}^N x_i W_i}{\sum_{i=1}^N W_i} = \frac{\sum_{i=1}^N x_i V_i}{\sum_{i=1}^N V_i} \quad (3)$$

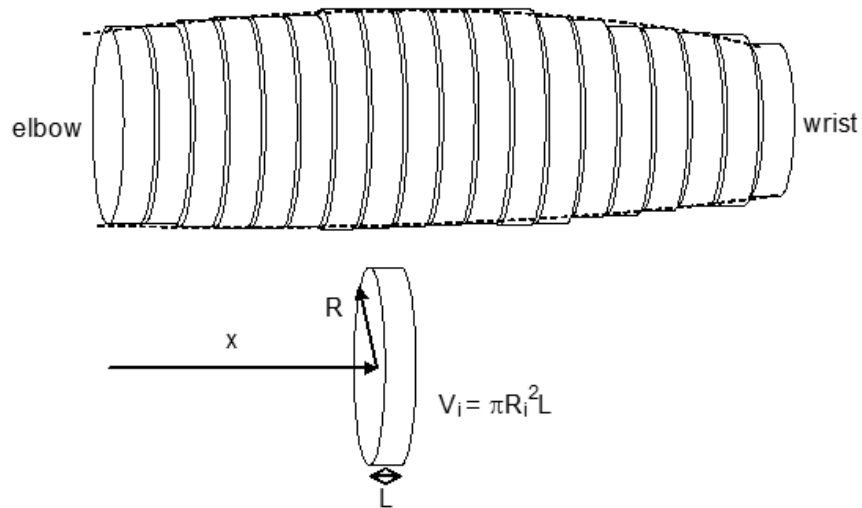


Figure 2. Segmenting the arm and weight each segment by its distance from the elbow.

Table 1. A female's center of gravity data and calculations.

Measurement	x_i (cm)	C_i (cm)	R_i (cm)	V_i (cm ³)	$V_i * x_i$ (cm ⁴)
1	0.5	23.6	3.756	44.32	22.16
2	1.5	23.4	3.724	43.568	65.352
3	2.5	23.1	3.676	42.452	106.13
4	3.5	22.9	3.645	41.739	146.087
5	4.5	22.4	3.565	39.927	179.672
6	5.5	22.1	3.517	38.859	213.725
7	6.5	22.1	3.517	38.859	252.584
8	7.5	21.8	3.47	37.828	283.71
9	8.5	21.6	3.438	37.133	315.631
10	9.5	21.3	3.39	36.103	342.979
11	10.5	20.6	3.279	33.778	354.669
12	11.5	19.8	3.151	31.192	358.708
13	12.5	18.8	2.992	28.124	351.55
14	13.5	18.3	2.913	26.658	359.883
15	14.5	17.5	2.785	24.367	353.322
16	15.5	16.8	2.674	22.463	348.177
17	16.5	16.3	2.594	21.139	348.794
18	17.5	15.5	2.467	19.12	334.6
19	18.5	15.2	2.419	18.383	340.086
20	19.5	14.7	2.34	17.202	335.439
21	20.5	14.5	2.308	16.735	343.068
22	21.5	14.2	2.26	16.046	344.989
23	22.5	14.2	2.26	16.046	361.035
24	23.5	14.5	2.308	16.735	393.273

A set of sample data for a female's forearm is shown in Table 1. The distance x_i is the distance from the elbow to the center of gravity of each 1 cm long segment. The sum of volumes in equation (3) can be found by adding all the values in column 5 and the sum of the products $x_i V_i$ can be found by adding all the values in column 6. Adding each of the last two columns gives totals of 709 cm³ for forearm volume and 6856 cm⁴ for the sum of the products of position and volume. Dividing the sum of the weighted segments by the total weight gives the center of gravity, 9.67cm.

Once students have completed their own calculations and found the center of gravity of their arm, compare and contrast the range of values found. How does each person's center of gravity position compare to the appearance of their forearm? What assumptions have been made about the forearm in this exercise? Note that you have assumed a constant density in the forearm, which is not quite true.

You may also wish to introduce anthropometric tables to your students. These tables represent average values for many, many people. The anthropometric table for center of gravity² is given in Table 2. Note that there are separate columns for men and women. To use this table, multiply the length of your forearm by the percentage given in the table.

For the sample data given earlier, one would multiply the forearm's length of 23.5 cm by 0.434 to get 10.2cm. Note that this value is close to the value of 9.67cm calculated earlier.

Table 2. Anthropometric Table: Segment Center of Gravity.

(Expressed as a Percentage of Segment Length, from proximal end)

Segment	Males	Females
Head and Neck	55.0	55.0
Whole Trunk	63.0	56.9
Thorax	56.7	56.3
Abdomen and Pelvis	44.5	39.0
Upper Arm	43.6	45.8
Forearm	43.0	43.4
Hand	46.8	46.8
Thigh	43.3	42.8
Leg	43.4	41.9
Foot	50.0	50.0

Acknowledgments:

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References:

1. J.D. Cutnell and K.W. Johnson, Physics, 4th ed. (John Wiley & Sons, Inc., New York, 1998), p. 247.
2. Kreighbaum, E., & Barthels, K. M., Biomechanics: A qualitative approach for studying human movement (Boston, MA, Allyn and Bacon, 1985), p. 655.